Analysis and Design of Class-\textit{E}_3\textit{F} and Transmission-Line Class-\textit{E}_3\textit{F}_2 Power Amplifiers

Mury Thian and Vincent F. Fusco, Fellow, IEEE

Abstract—In this paper, analysis and synthesis approach for two new variants within the Class-EF power amplifier (PA) family is elaborated. These amplifiers are classified here as Class-\textit{E}_3\textit{F} and transmission-line (TL) Class-\textit{E}_3\textit{F}_2. The proposed circuits offer means to alleviate some of the major issues faced by existing topologies such as substantial power losses due to the parasitic resistance of the large inductor in the Class–EF load network and deviation from ideal Class-EF operation due to the effect of device output inductance at high frequencies. Both lumped-element and transmission-line load networks for the Class-\textit{E}_3\textit{F} PA are described. The load networks of the Class-\textit{E}_3\textit{F} and TL Class-\textit{E}_3\textit{F}_2 amplifier topologies developed in this paper simultaneously satisfy the Class-EF optimum impedance requirements at fundamental frequency, second, and third harmonics as well as simultaneously providing matching to the circuit optimum load resistance for any prescribed system load resistance. Optimum circuit component values are analytically derived and validated by harmonic balance simulations. Trade-offs between circuit figures of merit and component values considering the limitations in practical implementation are discussed.

Index Terms—Class E, Class F, Class E/F, Class EF, Fourier series, harmonic tuning, high efficiency, high frequency, impedance matching, power amplifier, quality factor, switching amplifier, transistor circuits, transmission line.

I. INTRODUCTION

Class-E power amplifier (PA) design is based upon wave-shaping techniques [1]-[3] and Class-F PA design is based upon appropriate harmonic-termination [4]. Both schemes offer high efficiency at microwave and higher frequencies. The Class-E PA is attractive due to its simple load network, but it suffers from high peak switch voltage stress of 3.56\textit{V}_{\text{CC}} \text{ (or 3.65\textit{V}_{\text{CC}} in case of Class-E with parallel circuit [4]). This requirement can limit practical implementation even for moderate output power levels. On the other hand, the Class-F PA requires a much lower peak switch voltage of just 2\textit{V}_{\text{CC}}. However, it requires a complex load network which, for an ideal amplifier, is required to present infinite harmonic

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M. Thian and V. F. Fusco are with the Electronics, Communications, and Information Technology (ECIT) Institute, the Queen’s University Belfast, Queen’s Road, Queen’s Island, Belfast, BT3 9DT, UK (phone: +44-28-9097-1808; fax: +44-28-9097-1702; e-mail: m.thian@qub.ac.uk).

Fig. 1. Class-EF power amplifier as in [8]-[9]

Fig. 2. Class-EF$_2$ power amplifier with lumped-element load-network as in [10]-[11]

The inverse Class-E PA [5]-[6] requires 20% lower peak switch voltage than its Class-E counterpart but like Class-F and F$^{-1}$ PAs, its implementation is limited to low-to-medium power applications since it does not inherently have mechanism in place to absorb the device output capacitance associated with higher power devices.

A hybrid of Class-E and Class-F$^{-1}$, the so called Class-E/F, was proposed in [7]. It exhibits a trade-off between the circuit simplicity of Class E and the high performance of Class F$^{-1}$. Like in the Class-E topology, the device output capacitance can be incorporated into the circuit allowing the zero-voltage
switching (ZVS) and zero voltage-derivative switching (ZdVS) conditions required for soft-switching operation, (i.e. minimizing the switch current and voltage waveform overlaps in order to reduce power loss during switching transient). The Class-E/F family also allows increased tolerance to large transistor output capacitance, extending its maximum operating frequency ($f_{\text{MAX}}$) beyond that of the Class-E. However, the peak switch voltage is still rather high especially where low-voltage breakdown device technology like CMOS is to be used, i.e. it varies between 3.14V (Class F$^3$) and 3.56V (Class E) depending upon which harmonic-termination types (odd/even) and order are used.

A hybrid of the Class-E and Class-F, the so called Class-Φ in [8] or Class-ΦE in [9], Fig. 1, replaces the bulky RF choke inductance (RFC) in the classical Class-E and Class-F topologies with a quarter-wave transmission line ($\lambda/4$ TL), facilitating practical implementation at microwave frequencies. To avoid confusion, this type of PA will be called Class-ΦE in this paper. Experimentally reported in [8] and extensively analyzed in [9] the Class-ΦE PA benefits from low peak switch voltage of $2V_{\text{CC}}$ as in Class-F and at the same time offers soft-switching operation as in Class-E.

Like Class-ΦE, Class-DE can also operate at high frequencies with the same reduced peak switch voltage, but the circuit topology is more complex since it requires two transistors and an additional balun to provide anti-phase input signals for each transistor. A variant of Class-ΦE, introduced recently as Class-Φ2 in [10] or Class-ΕF2 in [11], Fig. 2, considers termination only of the first three harmonics. This finite harmonic-tuning strategy proves sufficient since higher harmonic tuning does not provide significant efficiency improvement. By replacing the $\lambda/4$ TL in the Class-ΕF circuit with a series-parallel resonator, Fig. 2, the class-ΕF2 amplifier can further increase the maximum operating frequency of the Class-ΕF PA for given power delivery, supply voltage, and device output capacitance.

Device output inductance predominately constituted of bond wire and device lead inductance has a major detrimental effect at high frequencies. These parasitic inductances are unfortunately not included in the analysis of the aforementioned Class-ΕF and its derivative topologies. In addition, the lumped-element load networks in Figs. 1 and 2 are not suitable for microwave and millimetre-wave applications. For example, series inductor $L_0$ in the resonant tank in Fig. 1 is typically of large value. This large inductance is accompanied with a large electrical series resistance (ESR) whose value is comparable to the optimum load resistance. This, in turn, will cause significant power losses that degrade overall amplifier efficiency. Large inductors also have low self-resonant frequency ($f_{\text{SR}}$) further limiting application at high frequencies. In Fig. 2, series-parallel tank circuits $L_1$, $C_1$, $L_2$, and $C_2$ provides simultaneous open circuit at fundamental and third harmonic as well as short circuit termination at the second harmonic. Although in theory this method of termination should be straightforward, in practice the lumped inductors and capacitors have, respectively, different inductance and capacitance values at different frequencies. In an even worse scenario, the reactance of a large inductor could easily be no longer inductive, i.e. it could become capacitive, at second and higher harmonic frequencies due to its low $f_{\text{SR}}$ characteristic and therefore simultaneous three-harmonic terminations are very challenging particularly at high microwave frequencies. In addition, the circuits reported in [8]-[11] does not provide a means for impedance transformation from the optimum resistance value to the system impedance which is typically 50Ω.

The novel Class-ΕF and transmission-line (TL) Class-ΕF2 amplifier topologies proposed in this paper overcome the afore-mentioned limitations of existing designs. The state-space approach used in the analysis in [11] requires optimum circuit component values be obtained by repeated numerical trial, and the numerical simulation approach offers little insight to the inter-relationship between components. In this paper, we offer a means for obtaining optimum circuit component values of the new Class-ΕF and TL Class-ΕF2 PAs using explicit equations.

Similar nomenclature to that introduced in [7] is adopted in this paper. In case of Class-F, subscripts refer to the even harmonic number (2, 4, 6, etc) at which the collector is short circuited. For example Class-F2 means short circuited only at the second harmonic. In case of Class-E the subscripts refer to the odd harmonic number (3, 5, 7, etc) at which the load network seen by the switch is simply a shunt capacitor $C$. The absence of a subscript means that the impedance requirements at all even (for Class-F) or odd (for Class-E) harmonic frequencies are met. The Class-ΕF amplifier, from its name, therefore satisfies short-circuit termination at all even harmonics as in Class-F but provides an open circuit only at the third harmonic rather than all odd harmonics (higher than $f_2$) as in Class-E. On the other hand, the Class-ΕF2 PA is designed to meet the third-harmonic impedance requirement of the Class-E and at the same time satisfy the second-harmonic requirement of the Class-F.

This paper is organized as follows: section II will give a brief introduction to the Class-ΕF PA and discuss design trade-offs in detail as a precursor to sections III and IV, section III will introduce the new Class-ΕF PA and explain its basic operation, section IV presents analytical derivation for the optimum circuit component values of the proposed transmission-line Class-ΕF2 PA including Fourier analysis to predict the switch voltage waveform, and finally section V gives design examples for both Class-ΕF2 and ΕF2 amplifiers which can be compared to Class-ΕF counterpart by means of harmonic-balance simulations.

II. CLASS-ΕF POWER AMPLIFIER

Unlike Class-E and Class-F, the Class-ΕF PA operates with duty ratio ($D$) of less than 50%. This means that the transistor is switched ON for a period of less than a half cycle. For convenience, a parameter namely dead time ($\tau_D$, in radians) is
used here rather than duty ratio and their relationship is given by 
\[ D = 0.5 - \tau_D/(2\pi). \] For example, \( \tau_D = \pi/4 \) (conduction angle is \( 180^\circ - 45^\circ = 135^\circ \)) results in \( D = 0.375 \). For a given supply voltage (\( V_{CC} \)), power throughput (\( P_{OUT} \)), operating frequency (\( f_0 \)), dead time, and loaded quality factor (\( Q_L \)), circuit component values in Fig. 1 can be calculated as follows, [9]:

\[
R = \frac{2 (1 + \cos \tau_D)^2 V_{CC}^2}{\pi^2 P_{OUT}} \tag{1}
\]

\[
C = \frac{\pi}{2} \left( \frac{\sin \tau_D}{1 + \cos \tau_D} \right)^2 \frac{P_{OUT}}{\omega_0 V_{CC}^2} \tag{2}
\]

\[
L = \frac{2 (1 + \cos \tau_D)^2 \tau_D - 0.5 \sin(2\tau_D)}{\pi^2 \sin^2 \tau_D} \frac{V_{CC}^2}{\omega_0 P_{OUT}} \tag{3}
\]

\[
L_0 = \frac{Q_L R}{\omega_0} \tag{4}
\]

\[
C_0 = \frac{1}{\omega_0 Q_L R} \tag{5}
\]

Normalized \( R, L, \) and \( C \) are plotted in Fig. 3 as a function of dead time. Maximum load resistance value is achieved at \( \tau_D = 0^\circ \) and its value is the same as in Class-F. Maximum inductance value is achieved at \( \tau_D = 64^\circ \) and its value is \( 0.375 V_{CC}^2/(\omega_0 P_{OUT}) \).

The maximum operating frequency is obtained from (2) where \( C \) now entirely represents device output capacitance (\( C_{OUT} \)).

\[
f_{MAX} = \frac{1}{4} \left( \frac{\sin \tau_D}{1 + \cos \tau_D} \right)^2 \frac{P_{OUT}}{C_{OUT} V_{CC}^2} \tag{6}
\]

Another figure of merit of the PA, namely power-output capability, gives information about what the voltage (\( V_{PK} \)) and current (\( I_{PK} \)) device stress is for a given power throughput. While peak switch voltage is \( 2V_{CC} \), the peak switch current and power-output capability are computed as follows:

\[
I_{PK} = \begin{cases} 
2I_s & ; \tau_D \leq \pi/2 \\
2I_s \sin \tau_D & ; \pi/2 < \tau_D < \pi 
\end{cases} \tag{7}
\]

where

\[
I_s = \frac{\pi}{1 + \cos \tau_D} I_0 \tag{8}
\]

\( I_0 \) is dc current.

Using (6) and (10), normalized \( f_{MAX} = f_{MAX} C_{OUT} V_{CC}^2/P_{OUT} \) and \( P_{MAX} \) are plotted in Fig. 4. In [8], the power-output capability of the Class-EF PA is claimed to be the same as that of Class-F, i.e. \( 0.5/\pi \approx 0.16 \). However, Fig. 4 shows that the power-output capability is not constant but is a function of dead time. From Fig. 4, it can be seen that \( P_{MAX} \approx 0.16 \) is only achieved at \( \tau_D = 0^\circ \). From (2) and (3) this would result in zero values for \( C \) and \( L \), reducing the Class-EF circuit in Fig. 1 to the Class-F circuit in [4]. The \( P_{MAX} \) for Class-E, \( 0.0981 \), is obtained at \( \tau_D = 48.5^\circ \). At this point, the \( f_{MAX} \) of Class-E is obtained at \( \tau_D = 75^\circ \). At this point, the \( P_{MAX} \) of the Class-EF PA i.e. 0.132 is 35% higher than Class-E and is even still higher than the Class-A and -B counterparts (0.125). Although the operation of the Class-EF amplifier can be extended to higher frequency by increasing the dead time, this would result in low \( P_{MAX} \) (Fig. 4) and yield
low load resistance values (Fig. 3) making matching to 50 Ω more difficult.

III. NOVEL CLASS-E$_3$F POWER AMPLIFIER

The series inductor ($L + L_0$) in the load network of the Class-EF circuit in Fig. 1 typically has a large value. A large printed spiral inductor in MMIC technology implies (1) high parasitic resistance reducing output power and consequently efficiency, (2) low self-resonant frequency (narrow inductor bandwidth), and (3) increased chip area. Ideally, the series circuit $L_0 - C_0$ tuned at fundamental frequency would provide an open circuit at higher harmonics. In practice, this is almost impossible to achieve since at slightly above the self-resonant frequency the inductor would no longer be inductive but capacitive. The fourth harmonic (for example) can already easily fall outside the bandwidth of a large inductor. This means that the series $L_0 - C_0$ circuit would satisfy the open-circuit requirement only up to third harmonic since at $4f_0$, the inductor is presenting capacitive characteristics. In this case, since only the third harmonic component is filtered by a practical $L_0 - C_0$ tank, not only the fundamental frequency signal but other higher odd harmonics ($5f_0, 7f_0$, etc) would also appear at the load resistance. Note that the ideal tank circuit in the Class-E PA effectively isolates only the odd harmonic signals ($3f_0, 5f_0$, etc) from the load resistance because the even harmonic signals are already short circuited by the $\lambda/4$ TL. As operating frequency increases, the practical tank circuit may only work effectively up to the second harmonic above which it can no longer provide an open circuit. In this case, since the $\lambda/4$ TL has short-circuited the second harmonic, the $L_0 - C_0$ tank in Fig. 1 can be removed obviating the aforementioned problem due to the large inductor requirement. However, since there is now no mechanism in place to filter odd harmonics ($3f_0, 5f_0$, etc) particularly the dominant $3f_0$ component, strong harmonic distortion at the output would deleteriously influence amplifier performance.

The novel Class-E$_3$F power amplifier introduced in this paper, Fig. 5(b), with its reduced synthesized inductance value and at the same time facilitating a means for output matching is now described. Unlike the series tank circuit in the Class-EF PA, the parallel tank in Class-E$_3$F PA does not need to be extremely broad band. The principal requirement for this tank is that it must work effectively up to third harmonic. This requirement is relatively easy to meet since the optimum inductor value ($L_3$) is much less than that of Class-EF PA ($L + L_0$) and therefore has typically wider bandwidth.

A. Class-E$_3$F PA with Lumped-Element Load-Network

The Class-EF circuit in Fig. 1 is not equipped with a means for an impedance transformation needed to match the optimum load resistance ($R$) to the system resistance ($R_L$). A simple L-type matching circuit comprised of $L_3$ and $C_3$ is shown in Fig. 5(a) where $R_L$ is typically larger than $R$. The values of $L_3$ and $C_3$ are calculated as follows:

$$L_3 = \frac{R}{\omega_0^2} \sqrt{\frac{R_L}{R} - 1}$$  \hspace{1cm} (11)

$$C_3 = \frac{1}{\omega_0^2 R_L} \sqrt{\frac{R_L}{R} - 1}$$  \hspace{1cm} (12)

The proposed Class-E$_3$F circuit is depicted in Fig. 5(b). As in Class-EF PA, all even harmonic components appearing at the collector of the transistor are shorted by the $\lambda/4$ TL, leaving only the odd harmonic components and consequently producing a square-wave collector voltage. Parallel resonator $L_3 - C_3$ is tuned at $3f_0$ so as to provide an open-circuit termination. At $f_0$, this resonator behaves like an inductor $L_{EQ}$ whose value is $9/8 L_3$. This $L_{EQ}$ can be designed to represent the total inductance of $L$ and $L_3$ in Fig. 5(a). Since the square-wave voltage at the collector is entirely preserved as in Class-F operation and an open-circuit requirement of the Class-E mode is met only at the third harmonic, this topology is called Class-E$_3$F. The values of $L_3$ and $C_3$ are calculated as follows:

$$L_3 = \frac{8}{9} (L + L_1)$$  \hspace{1cm} (13)

$$C_3 = \frac{1}{9\omega_0^2 L_3}$$  \hspace{1cm} (14)

B. Class-E$_3$F PA with Transmission-Line Load-Network

The transmission-line implementation of the Class-E$_3$F PA in Fig. 5(b) is presented in Fig. 6. The role of the parallel-resonator $L_3 - C_3$ in Fig. 5(b) to provide an open circuit at $3f_0$. 
is taken over by TL1 and TL2. The electrical lengths of both series line TL1 and open-circuit shunt stub TL2 are 30° at $f_0$. At $3f_0$, the node that connects to TL1, TL2, and $C_b$ is grounded. This grounded node is subsequently transformed into an open circuit by TL1. At $f_b$, TL1 in corporation with TL2 transform the system impedance $R_L$ into the optimum impedance required for Class-EF operation, i.e. $R + j\omega L$, where $R$ and $L$ are given in (1) and (3), respectively. The characteristic admittance of TL1 and TL2, i.e. $Y_1$ and $Y_2$, can be found by solving these two equations simultaneously:

$$ G \left( Y_1 - \frac{Y_1}{3} \right)^2 + G_2 \frac{Y_1}{3} = \frac{4G_1}{3} Y_1^2 \quad (15) $$

$$ B \left( Y_1 - \frac{Y_1}{3} \right)^2 + G \left( Y_1 + Y_2 \right) \left( Y_1 - \frac{Y_1}{3} \right) - G_2 \frac{Y_1}{\sqrt{3}} \quad (16) $$

where

$$ G = \frac{R}{R^2 + \omega^2 L^2} \quad (17) $$

$$ B = \frac{-\omega L}{R^2 + \omega^2 L^2} \quad (18) $$

In the transmission-line Class-E PA [4], the dc-bias $\lambda/4$ TL is placed between TL2 and $C_b$ rather than at the collector as in Fig. 6 since the Class-E mode requires second-harmonic peaking (open circuit) rather than second-harmonic short-circuit termination at the collector as in Class-EF PA. As a result, the electrical length of TL1 can be shortened to compensate for the detrimental effects of the device output inductance. Unfortunately, this strategy cannot be adopted for the transmission-line Class-E$_3$F PA shown in Fig. 6. The simple topology in Fig. 6 is useful when the device output inductance is considerably small and the operating frequency is not very high. Otherwise, a new topology discussed in the next section can be utilized.

IV. TRANSMISSION-LINE CLASS-E$_3$F$_2$ POWER AMPLIFIER

The maximum achievable efficiencies for incremental inclusive control of harmonics from the first to the fifth calculated in [12] are, respectively, 50%, 70.7%, 81.6%, 86.6%, and 90.5%. These results show that, theoretically, proper second and third harmonic terminations have the largest impact on efficiency enhancement, whereas the influence of higher harmonics decreases with increasing harmonic order. The Class-E$_3$F$_2$ PA analyzed here controls only the first three harmonics of the Class-EF PA yet as will be demonstrated later it is still capable of delivering high efficiency.

Next, a Class-E$_3$F$_2$ power amplifier with transmission-line load-network is shown in Fig. 7. Unlike to-date existing circuits, this topology incorporates a series inductance $L$ so as to eliminate the effect that the parasitic device output inductance (bond wire and lead package inductance) has on the amplifier’s performance particularly at high frequencies. The proposed circuit is also capable of minimizing power losses due to the ESR of the large series inductors required in the load networks of circuits in Figs. 1 and 2. As will be shown later in section V, this ESR can substantially degrade efficiency since its value is comparable to that of the load resistance.

![Fig. 6. Transmission-line Class-E$_3$F PA power amplifier](image)

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![Fig. 7. Novel Class-E$_3$F$_2$ power amplifier with transmission-line load-network](image)

Fig. 7. Novel Class-E$_3$F$_2$ power amplifier with transmission-line load-network

The TL Class-E$_3$F$_2$ PA is designed to be able to simultaneously satisfy Class-EF impedance load requirements at fundamental, second, and third harmonic frequencies. The impedances seen by the transistor (including the shunt capacitor $C$) would be $R + j\omega L$ at fundamental frequency, zero (short circuit) at second harmonic, and infinite (open circuit) at third harmonic. At the same time, the TL synthesis presented here also enables impedance transformation from $R_L$ to any system impedance $R_L$ (typically 50 Ω).

A. Basic Operation

The equivalent circuits for the TL Class-E$_3$F$_2$ PA are illustrated in Fig. 8, and $C_b$ is a dc blocking/bypass capacitor. The electrical lengths of TL1 and TL2, $\theta_1$ and $\theta_2$, in Fig. 7 are 30° at $f_0$ or equivalently 90° at $3f_0$. At the third harmonic frequency, the open-circuit shunt stub TL2 short circuits any impedance located to its right-hand side (i.e. TL3, $C_b$, $R_L$) and, subsequently, the series TL1 transforms this short circuit impedance into an open circuit, Fig. 8(c).

At $2f_0$, the $\lambda/4$ TL short circuits any impedance located to its right hand side (i.e. $C_b$, $R_L$). Short-circuited TL3 behaves like
an inductor \((L_3)\) to ground while open circuit stub TL2 with electrical length of 60° behaves like a capacitor \((C_2)\) to the ground (bear in mind that in order to behave as an inductor at 2\(f_0\) the electrical length of TL3 must be less than 90° at 2\(f_0\) or less than 45° at \(f_0\)). This parallel \(L_3 - C_2\) circuit designed to resonate at 2\(f_0\) behaves like a capacitor \((C_2)\) to the ground (bear in mind that in order to behave as an inductor at 2\(f_0\) the electrical length of TL3 must be less than 90° at 2\(f_0\) or less than 45° at \(f_0\)). This parallel \(L_3 - C_2\) circuit designed to resonate at 2\(f_0\) thus presents an open-circuit termination to TL1. Open-circuited TL1 with electrical length of 60° behaves like a capacitor \((C_1)\). The required short-circuit termination at second harmonic for the Class-EF operation is provided by the series \(L - C_1\) circuit, Fig. 8(b).

\[
(2\omega_b) L_1 C_1 = 1 \tag{19}
\]

where

\[
L_1 = \frac{Z_1 \tan(2\theta_1)}{2\omega_b} \tag{20}
\]

\[
C_1 = \frac{\tan(2\theta_1)}{2\omega_b Z_1} = \frac{\tan(\pi/3)}{2\omega_b Z_1} = \frac{\sqrt{3}}{2\omega_b Z_1} \tag{21}
\]

and

\[
(2\omega_b) L_3 C_2 = 1 \tag{22}
\]

where

\[
C_2 = \frac{\tan(2\theta_1)}{2\omega_b Z_2} = \frac{\tan(\pi/3)}{2\omega_b Z_2} = \frac{\sqrt{3}}{2\omega_b Z_2} \tag{23}
\]

Substitutions (20)-(21) into (19) result in

\[
Z_1 = \sqrt{3} Z_1 \tan(2\theta_1) \tag{24}
\]

and substitution (23) into (22) yields

\[
Z_2 = 2\sqrt{3} \omega_b L \tag{25}
\]

where \(Z_{11}, Z_{21}\), and \(Z_{23}\) are, respectively, the characteristic impedances of TL1, TL2, and TL3.

Consider Fig. 8(a) for the equivalent circuit at fundamental frequency. TL3 transforms the resistance \(R_L\) to admittance \(Y_{IN1} = G_{IN1} + jB_{IN1}\) where \(G_{IN1}\) and \(B_{IN1}\) are given in (26) and (27).

\[
G_{IN1} = \frac{G_i Y_i^2 \sec^2 \theta_i}{Y_i + (G_i \tan \theta_i)^2} \tag{26}
\]

\[
B_{IN1} = \frac{Y_i \tan \theta_i (Y_i + G_i^2)}{Y_i + (G_i \tan \theta_i)^2} \tag{27}
\]

where \(G_i = 1/R_L\) and \(Y_i = 1/Z_i\).

Open-circuit stub TL2 and \(Y_{IN1}\) form \(Y_{IN2} = G_{IN2} + jB_{IN2}\)

\[
G_{IN2} = G_{IN1} \tag{28}
\]

\[
B_{IN2} = B_{IN1} + \frac{\tan(\pi/6)}{Z_2} \tag{29}
\]

Substitution (24) into (29) yields

\[
B_{IN2} = B_{IN1} + \frac{Y_i}{3 \tan(2\theta_i)} \tag{30}
\]

Subsequently, TL1 transforms the admittance \(Y_{IN2}\) to the load resistance \(R\) as required for Class-EF operation mode. The two unknown variables \(Z_3\) and \(\theta_3\) can be solved using (31)-(32) while \(Z_1\) can be computed using (25).

\[
\frac{G_{IN1} Y_i^2 \sec^2 \theta_i}{(Y_i - B_{IN2} \tan \theta_i)^2 + (G_{IN2} \tan \theta_i)^2} = \frac{1}{R} \tag{31}
\]

\[
(Y_i \tan \theta_i + B_{IN2})(Y_i - B_{IN2} \tan \theta_i) = G_{IN2}^2 \tan \theta_i \tag{32}
\]

where \(Y_i = 1/Z_i\).

Fig. 8. Equivalent circuits of the transmission-line Class-EF PA at (a) fundamental frequency (b) second harmonic (c) third harmonic

B. Switch-Voltage Waveform

The ideal voltage square waveform at the collector of the Class-EF PA will not be obtained in the Class-EF PA since only the second-harmonic component rather than all even harmonics is shorted. The relationships between the peak switch voltage, harmonic-termination order, and the dead time are investigated in this section.
First, consider the switch voltage equation of the Class-EF PA for a full RF cycle in (33). Using Fourier series, this equation can be represented as a sum of dc component, fundamental-frequency component, and infinite odd multiples of fundamental-frequency components, (34). For even multiples of the fundamental frequency \((n = 2, 4, 6, \text{ etc})\), the values of \(a_n\) and \(b_n\) are zero since a short-circuit condition is enforced by the shorted \(\lambda/4\) TL. Using Fourier integral, the values of \(a_n\) and \(b_n\) can be calculated. Here, up to fifth harmonic components, (35)-(40), are considered since higher harmonic tuning does not provide significant advantage.

\[
v_{sw}(\omega t) = \begin{cases} 
0 & \text{; } 0 \leq \omega t \leq \pi - \tau_d \\
\frac{-I_s(\cos \tau_d + \cos \omega t)}{\omega C} & \pi - \tau_d \leq \omega t \leq \pi \\
\frac{2V_{cc}}{\omega C}(\cos \tau_d - \cos \omega t) + 2V_{cc} & \pi \leq \omega t \leq 2\pi - \tau_d \\
\frac{I_s}{\omega C}(\cos \tau_d - \cos \omega t) + 2V_{cc} & 2\pi - \tau_d \leq \omega t \leq 2\pi 
\end{cases}
\]  
\tag{33}

\[
v_{sw}(\omega t) = V_{cc} + \frac{2V_{cc}}{\pi} \sum_{n=1}^{\infty} \left\{ a_n \sin(n\omega t) + b_n \cos(n\omega t) \right\}
\]  
\tag{34}

\[
a_n = 1 + \cos \tau_d
\]  
\tag{35}

\[
b_n = \frac{\tau_d - 0.5\sin(2\tau_d)}{1 - \cos \tau_d}
\]  
\tag{36}

\[
a_n = \frac{(1 + \cos \tau_d)(\cos(2\tau_d))}{3}
\]  
\tag{37}

\[
b_n = \frac{(1 + \cos \tau_d)\sin(2\tau_d)}{3}
\]  
\tag{38}

\[
a_n = \frac{(1 + \cos \tau_d)(16\cos^4 \tau_d - 14\cos^2 \tau_d + 1)}{15}
\]  
\tag{39}

\[
b_n = \frac{(1 + \cos \tau_d)(8\cos^2 \tau_d - 3)\sin(2\tau_d)}{15}
\]  
\tag{40}

Fig. 9 depicts switch voltage waveforms for finite harmonic tuning for several dead time values. The collector voltage peak is not the mirror-image plateau of the Class-EF PA. Instead, ripple arises as it must where the harmonic content is limited. For the same reason, the switch voltage displays approximate half-wave symmetry, a property of a shorted \(\lambda/4\) TL [8]. As more harmonic tuning is applied this results in more ripple around the peak. Interestingly, the peak-to-peak of the ripple is reduced as the dead time is increased.

V. DESIGN AND VERIFICATION

To validate the proposed Class-E\(_3\)F PA and also the TL load-network synthesis of the Class-E\(_3\)F PA, simulations are performed within Agilent’s Advanced Design System (ADS). The design goal is to synthesize a 27 dBm power amplifier operating from a 5 V dc supply voltage. The design frequency is 2.5 GHz and the dead time of 45° is arbitrarily chosen.

First, the values of \(R, L, \text{ and } C\) are calculated using (1)-(3). For the lumped-element Class-E\(_3\)F design, Fig. 5(b), the other circuit component values \(C_1, L_3, \text{ and } C_3\) are calculated using (12)-(14). The characteristic impedances of TL1 and TL2 of the transmission-line Class-E\(_3\)F PA in Fig. 6 are computed using (15)-(18) and their values are, respectively, 86.6 Ω and 36.6 Ω. For the TL Class-E\(_3\)F design, Fig. 7, the characteristic impedance of TL1 (\(Z_j\)) is computed using (25). The electrical length and characteristic impedance of TL3 are obtained by solving (31)-(32). Once \(Z_j\) and \(\theta\) have been found, \(Z_j\) is calculated using (24). The circuit component values for both Class-E\(_3\)F and TL Class-E\(_3\)F PAs are presented in Table I.
Smith Charts in Fig. 10 depict simulated load impedances at the first three harmonics. Short-circuit and open-circuit terminations at, respectively, 5 GHz and 7.5 GHz are satisfactorily met and the required fundamental-frequency impedance is too.

The results of harmonic-balance simulation performed in order to compute output power, dc current, and peak switch current are presented in Table I. The peak switch currents of 362 mA for the lumped-element Class-E\textsubscript{F} and 380 mA for the TL Class-E\textsubscript{F2} agree well with the theoretical prediction of 368 mA computed using (7)-(8). In contrast, it is interesting to see that the simulated peak switch current in Fig. 10(b) in [9] deviates from the theoretical prediction by a factor of 2.

The second-, third-, and fourth-harmonic contents are, as expected, highly attenuated at the output port of the lumped-element Class-E\textsubscript{F}, TL Class-E\textsubscript{F}, and TL Class-E\textsubscript{F2} PAs. However, it is observed from the power spectra that the TL Class-E\textsubscript{F} PA, when compared to the other two topologies, generates the highest fifth-harmonic level, producing the most distorted sinusoidal output signal.

Voltage and current waveforms of the simulated Class-E\textsubscript{F} and Class-E\textsubscript{F2} circuits are, respectively, shown in Figs. 11 and 12. From Fig. 11, it can be observed that unlike in Class-E\textsubscript{F}, the transmission-line current does not replicate the load current during the interval of $\pi \leq \omega t \leq 2\pi - \tau_D$ since it is formed by two currents, i.e. the current goes to $C_1$ and the load current. In Fig. 12, ripple at the peak of switch voltage not seen in the Class-E\textsubscript{F} and Class-E\textsubscript{F2} PAs are generated since only the second harmonic component of the collector voltage is shorted. This ripple is approximately symmetrical about 10 V (2$V_{CC}$). Notice that the switch current is very similar to the Class-E\textsubscript{F} counterpart but other currents (mainly $i_C$) depart creating the ripple.

For a comparison, the Class-E\textsubscript{F} PA described in [8]-[9] is also designed for the same PA specification. The component values are computed using (1)-(5) with loaded $Q_L = 10$ and presented in Table I. The series inductors, $L$ and $L_0$, in the load network have total inductance value of almost 33 nH. In RFIC and MMIC implementations, the unloaded $Q$ factor of a spiral inductor typically ranges between 10 and 30 at microwave frequencies. Typically, the larger the inductance value the poorer the quality factor. Assuming the $Q$ factor of the spiral inductor is 20, this would be accompanied by an ESR of about 26 $\Omega$ whose value is comparable to the optimum load resistance $R$ of 29.5 $\Omega$. This would cause substantial efficiency degradation. In addition, large inductor has typically low self-
resonant frequency limiting its application at high frequencies. The steady-state current and voltage waveforms are given in Fig. 13 where maximally flat peak switch voltage of 2\(V_{CC}\) as in the new Class-E\(_2\)F PA and peak switch current of 376 mA are observed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Class E(_2)F</th>
<th>Transmission-line Class E(_2)F</th>
<th>Class EF ((Q_L = 10))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_s = 50 , \Omega)</td>
<td>(R_s = 50 , \Omega)</td>
<td>(R = 29.5 , \Omega)</td>
<td></td>
</tr>
<tr>
<td>(C = 0.35 , \text{pF})</td>
<td>(L = 1.1 , \text{nH})</td>
<td>(L = 1.1 , \text{nH})</td>
<td></td>
</tr>
<tr>
<td>(L_{3} = 2.4 , \text{nH})</td>
<td>TL1: 30(^\circ), 58 (\Omega)</td>
<td>(L_0 = 31.8 , \text{nH})</td>
<td></td>
</tr>
<tr>
<td>(C_3 = 0.2 , \text{pF})</td>
<td>TL2: 30(^\circ), 41 (\Omega)</td>
<td>(C_0 = 0.15 , \text{pF})</td>
<td></td>
</tr>
<tr>
<td>(C_{3}' = 1.1 , \text{pF})</td>
<td>TL3: 10(^\circ), 65 (\Omega)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**VI. CONCLUSION**

As design frequency increases, the practical implementation of the lumped-element load network of the Class-EF in [8]-[9] and Class-E\(_2\)F in [10]-[11] struggles to simultaneously satisfy the load-impedance requirements at fundamental frequency, second, and third harmonics. Transmission-line approaches for the synthesis of the Class-E\(_2\)F presented in this paper can elegantly eliminate this problem. In addition, the parasitic device-output inductances whose effects are significant at high frequencies are naturally incorporated within the new topology and thus deviation from the idealized operation of the Class-EF mode can be minimized.

A novel yet simple Class-E\(_2\)F PA was also presented to overcome the practical limitations of existing topologies due to the large values of series inductance typically required in the load network while still allowing the benefit of low switch voltage stress of just 2\(V_{CC}\). Finite harmonic tuning applied to the proposed circuits does not change the idealized Class-EF switch-current waveform. Unlike in Class-E\(_2\)F PA, the idealized Class-E\(_2\)F switch voltage waveform is altered in the TL Class-E\(_2\)F amplifier where ripple is observed at its peak. This is because even harmonic components higher than 2\(f_0\) are not short circuited at the collector. However, from the simulation results, it is observed that finite harmonic tuning up to the third harmonic is sufficient to facilitate high efficiency with minimal switch current and voltage overlaps.

**REFERENCES**


**Mury Thian** received his B.Sc. degree (cum laude) from Atma Jaya University, Jakarta, Indonesia, M.Sc. degree from Delft University of Technology, The Netherlands, and Ph.D. degree from the Queen’s University of Belfast, UK, all in electronics engineering. He joined the University of Birmingham, UK, as a Research Fellow before he returned to the Queen’s University of Belfast in 2009 working on radio technologies for short range Gigabit wireless communications.

He gained industrial experience in Philips Semiconductors, Nijmegen, the Netherlands in 2003-2004. His current research interests include MMIC design, micro-power circuits design, emerging device technology, and efficiency enhancement and linearization techniques for power amplifiers.

**Vincent F. Fusco** (S’82-M’82-SM’96-F’04) obtained his Bachelors degree in Electrical and Electronic Engineering (First Class Honours), PhD and DSc in Microwave Electronics from the Queen’s University of Belfast, where currently he holds a personal chair in High Frequency Electronic Engineering. Professor Fusco’s research interests include active antenna and front-end MMIC techniques. He is head of the High Frequency Laboratories at QUB where he is also director of the International Centre for System on Chip for Advanced Micro-wireless.

Professor Fusco has published over 350 scientific papers in major journals and in refereed international conferences. He has authored two text books, holds patents related to self-tracking antennas and has contributed invited chapters to several books. He is a Fellow of both the Institution of Engineering and Technology and the Institute of Electrical and Electronic Engineers. In addition he is a Fellow of the Royal Academy of Engineers and a member of the Royal Irish Academy. In 1986 he was awarded a British Telecom Communications Fellowship and in 1997 he was awarded the NI Engineering Federation Trophy for outstanding industrially relevant research.